(1)

المحاضرة الأخيرة

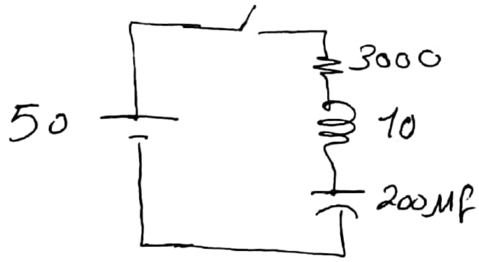
1- Series RLC (Time Domain)

2- Some examples using Laplace (F-Domain)

Example (1) Series RLC, $R = 3000\,\Omega$, $L = 10H$, $C = 200\mu f$, connected with DC battery 50v, through "S" switch, closed at $t=0$, with no initial charge.

Solution

using KVL



$$50 = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

تفاضل المعادلة لوجود تكامل

$$\therefore\ 0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i$$

$$0 = \frac{R}{L}\frac{di}{dt} + \frac{d^2i}{dt} + \frac{1}{LC}i$$

$$\therefore\ (D^2 + \frac{R}{L}D + \frac{1}{LC})i = 0 \rightarrow m_{1,2} = -\frac{R}{2L} \pm \sqrt{(\frac{R}{2L})^2 - (\frac{1}{LC})}$$

$$\therefore\ (D^2 + 300D + 500)i = 0$$

معادلة تفاضلية من الدرجة الثانية ليها تعويض للجذر

$-298.3 = m_1$

$-1.67 = m_2$

الجذر يتم تحليلها

وقفت المحاضرة هنا

real & Unequal (overdamped)

$$(\frac{R}{2L})^2 > \frac{1}{LC}$$

$$i = K_1e^{m_1t} + K_2e^{m_2t}$$

real, equal critical damping

$$(\frac{R}{2L})^2 = \frac{1}{LC}$$

$$i = e^{mt}(C_1 + C_2t)$$

Complex Underdamping

$$(\frac{R}{2L})^2 < \frac{1}{LC}$$

$$i = e^{\alpha t}[A_1\cos wt + A_2\sin wt]$$

where

$$\alpha = \frac{R}{2L}$$

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نرجع للمعادلة

$m_1 = -298.3$ ($\alpha+\beta$) , $m_2 = -1.67$ ($\alpha-\beta$)

$$\therefore i_1 = C_1 e^{-298.3t} + C_2 e^{-1.67t}$$

(1) to **Find Constants**

at $t=0$ $i_1 = 0$ (الوجود الملف) $\rightarrow$

$$\boxed{0 = C_1 + C_2} \rightarrow (1)$$

في نفس الوقت نعوض بـ t=0 في المعادلة الاصلية قبل التفاضل

(2) تفاضل المعادلة (1)

$$\therefore \frac{di_1}{dt} = -298.3 C_1 e^{-298.3t} + (-1.67) C_2 e^{-1.67t} \rightarrow (2)$$

(3) نرجع للمعادلة الاصلية قبل التفاضل KVL ونعوض عند $t=0$ ...

$$\therefore 50 = 300\cancel{0} i + 10 \frac{di}{dt} + \frac{1}{200\times10^{-6}} \int i\,dt$$

$\rightarrow 0$ $\rightarrow 0$

$$\therefore 5 = \frac{di}{dt} \rightarrow (3)$$

نرجع نعوض من (3) في (2) ونعوض بالرقم $t = 0$

$$\therefore 5 = -298.3 C_1 e^{0} + (-1.67) C_2 e^{0}$$

$$\therefore \boxed{4} \Rightarrow 5 = -298.3 C_1 - 1.67 C_2$$

نحل (1) و (4) نلاقي

$-0.0168 = C_1$

$0.168 = C_2$

$$\therefore \boxed{i_1 = 0.0168 e^{-1.67t} - 0.0168 e^{-298.3t}}$$

Max Current at $\frac{di}{dt} = 0$ ∴ نفاضل المعادلة ونساوي بالصفر

$$0.0175 = t \Leftarrow \frac{di_1}{dt} = 0 \therefore (-1.67)(0.0168) e^{-1.67t} - 0.0168 e^{-298.3t} (-298.3)$$

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$\boxed{2}$ Laplace

طريقة تستخدم لتحويل المعادلات للحل من الصورة التفاضلية إلى
صورة S Domain (Freq.) من خلال شوية قواعد كده

$\boxed{1}$ $i(t) \underset{}{\overset{\text{تتحول إلى}}{\rightleftharpoons}} I(s)$ ✓ مهم

في المقاوم: $V(t) \rightleftharpoons V(S)$

$\boxed{2}$ في الملف coil:

$i(t) \rightleftharpoons SI(s) - I(0)$ ✓ مهم → initial

$V(t) = L\dfrac{di}{dt} \Longrightarrow V(s) = SLI(s) - L\,I(0)$

$= L(SI(s) - I(0))$

$V_{bat} + V_{initial} = IR$

$V_{bat} = IR - V_{in}$

$I(0)$

$\boxed{3}$ في المكثف capacitor:

$V(t) = \dfrac{1}{C}\int i\,dt \pm V_{initial}$ مهم

$V(S) = \dfrac{1}{CS} I(s) \pm \dfrac{V(0)}{S}$

لو عكس الاتجاه minus

مع حفظ الجدول

① $\dfrac{A}{S} \longrightarrow A$

② $\dfrac{A}{S^2} = At$

③ $\dfrac{1}{S+a} = e^{-at}$

④ $\dfrac{1}{(S+a)^2} \Rightarrow te^{-at}$

⑤ $\dfrac{\omega}{S^2+\omega^2} \Rightarrow \sin(\omega t)$

⑥ $\dfrac{S}{S^2+\omega^2} = \cos\omega t$

⑦ $\dfrac{di}{dt} = SI(S) - I(0)$

⑧ $\dfrac{1}{C}\int i\,dt = \dfrac{I(S)}{CS} + \dfrac{V(0)}{S}$

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EX (2) Solve $i(t)$ using laplace for source. $V = 50\,V$

$\cdot\ V = 50e^{-100t}$

$$50 = 10i + 0.2\frac{di}{dt}$$

Take laplace

$$\frac{50}{S} = 10I(s) + 0.2\left(sI(s) - I(0)\right)$$

(initial) [illegible] 0.2 I(0)

at $t = 0 \rightarrow i = 0$ ما دام مفيش شرط ابتدائي

$$\therefore \frac{50}{S} = 10I(s) + 0.2\,S\,I(s) = I(s) = 10 + 0.2S$$

$$\therefore I(s) = \frac{50}{S(10+0.2S)} = \frac{A}{S} + \frac{B}{10+0.2S}$$

$$I(s) = \frac{A(10+0.2S)+BS}{S(10+0.2S)} = \frac{10A + S(B+0.2A)}{S(10+0.2S)}$$

$$\therefore 50 = 10A + S(B+0.2A)$$

$$\therefore 50 = 10A \quad \therefore A = 5$$

$$B + 0.2A = 0 \quad \therefore B = -0.2A = -1$$

$$\therefore I(s) = \frac{5}{S} - \frac{1}{10+0.2S} = \frac{5}{S} - \frac{5}{S+50}$$

ضربنا 0.2 عامل مشترك

$$\therefore I(t) = 5 - 5e^{-50t} = 5(1-e^{-50t})$$

(5)

EX(2) q5

Rl + ac

if supply was $(50\, e^{-100t}) \longrightarrow$ AC

$$50\, e^{-100t} = 10\, i + 0.2 \frac{di}{dt}$$

$$\frac{50}{S+100} = 10 I(s) + 0.2\left(S I(s) - I(0)\right)$$

$$= I(s)\left[10 + 0.2s\right]$$

$$I(s) = \frac{50}{(s+100)(10+0.2s)} = \frac{\cancel{50}\ 250}{(s+100)\cancel{(0.2)}(s+50)}$$

$$I(s) = \frac{A}{s+100} + \frac{B}{s+50} = \frac{250}{(s+100)(s+50)}$$

$\therefore$ $A(s+50) + B(s+100) = 250$

$\therefore$ $s(A+B) = 0$ $\qquad \therefore A = -B$

$50A + 100B = 250$

$\therefore$ $50B = 250$ $\qquad \therefore B = 5$, $A = -5$

$$\therefore I(s) = \frac{-5}{s+100} + \frac{5}{s+50}$$

$$\therefore I(t) = -5e^{-100t} + 5e^{-50t} \quad \#$$

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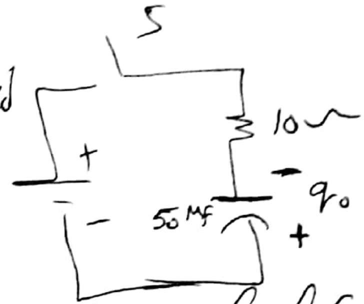
Ex(3)

RC + DC



in RC series ckt, capacitor is initially charged with $Q = 2500\,\mu C$, at $t=0$ switch closed, a constant supply 100V applied to the circuit, Find the current using laplace.

Sol.

$$100 = 10\,i + \frac{1}{C}\int i\,dt = 10\,i + \frac{1}{50\times10^{-6}}\int i\,dt$$

take laplace

$$\frac{100}{S} = 10\,I(S) + \left[\frac{I(S)}{CS} - \frac{V_{initial}}{S}\right]$$

$V_{initial} \rightarrow \frac{Q}{C}$

المعروف لو عندك اتنين تيار : $V_{initial}$ + V_{supply} في نفس الاتجاه = $I[Z]$

:. هصور الـ $V_{initial}$ الناحية التانية = بالسالب.

$$\frac{100}{S} = 10\,I(S) + \left[\frac{I(S)}{50\times10^{-6}S}\right] - \frac{2500\times10^{-6}}{50\times10^{-6}S}$$

$$\therefore I(S) = \frac{150}{10S + 2\times10^{4}} = \frac{15}{S + 2\times10^{3}}$$

$$\therefore \mathcal{L}^{-1}(I(S)) = I(t)$$

$$\therefore I(t) = 15\,e^{-2\times10^{3}t}$$